


Claims

1  In a multicarrier communication system in which a signal to be transmitted comprises
2
3 data bits to be converted into a symbol modulated by each subcarrier of the signal prior to
4 transmission on a channel, a method for minimizing a peak to average power ratio while
5 minimizing introduction of errors into the signal to be transmitted:
6 sampling the symbols to be transmitted of a frame;
7 compare magnitudes of the samples of the frame to a predetermined threshold to
8 determine whether sample magnitudes in the frame violate the
9 predetermined threshold, the predetermined threshold being selectable to
10 control the number of samples violating the threshold;
11 responsive to determining a sample magnitude does violate the predetermined
12 threshold, applying a differentiable penalty function to the samples having
13 magnitudes exceeding the predetermined threshold;
14 computing a net penalty function value, the net penalty function value responsive
15 to the individual penalty function values computed for the samples having
16 magnitudes exceeding the predetermined threshold;
17 computing a gradient vector responsive to the net penalty function value;
18 determining a direction of the gradient vector;
19 determining an upper limit correction value for each symbol, the upper limit
20 correction value being selectable to control an amount of signal to noise
21 ratio deterioration;

22 applying a correction to the symbols to be transmitted in a direction opposite to
23 the direction of the gradient vector of a magnitude not exceeding the
24 determined correction values for each symbol; and
transmitting the corrected symbols to the channel.

1 2. The method of claim 1 wherein determining an upper limit correction value for each
2 symbol, the upper limit correction value further comprises:

3 computing an interpoint distance between symbols;

4 selecting a correction value for a symbol as a value less than the interpoint

5 distance to ensure that the symbol is not mistaken for other symbols.

1 3. The method of claim 1 wherein applying a differentiable penalty function to the samples
2 having magnitudes exceeding the predetermined threshold comprises:

3 applying the function:

4

$$h(x[k]) = \begin{cases} (x[k] - T)^{2m} & \text{if } x[k] \geq T \\ 0 & \text{if } |x[k]| \leq T \\ (x[k] + T)^{2m} & \text{if } x[k] \leq -T \end{cases}$$

5 where m is a positive integer that decides the severity of penalty, T is the

6 predetermined threshold, x is the frame of data symbols expressed by: X =

7 $(r_0, r_1 \exp(j\theta_1), r_2 \exp(j\theta_2), \dots, r_{N/2-1} \exp(j\theta_{N/2-1}), r_{N/2})$, where r_i and θ_i denote

8 the magnitude and phase of symbol in channel i, and k is the number of

9 the symbol.
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1 4. The method of claim 3 wherein the net penalty function comprises:

$$f(x) = \sum_{k=0}^{N-1} h(x[k])$$

5. The method of claim 4, wherein the gradient vector is computed as:

$$\begin{aligned} \frac{\partial f}{\partial r_i} &= \sum_{k=0}^{N-1} \frac{dh(x[k])}{dx[k]} \cos\left(\frac{2\pi ki}{N} + \theta_i\right); i \in \{1, \dots, N/2 - 1\} \\ \frac{\partial f}{\partial r_0} &= \sum_{k=0}^{N-1} \frac{dh(x[k])}{dx[k]}; \frac{\partial f}{\partial r_{N/2}} = \sum_{k=0}^{N-1} \frac{dh(x[k])}{dx[k]} \cos(\pi k) \\ \frac{\partial f}{\partial \theta_i} &= -r_i \sum_{k=0}^{N-1} \frac{dh(x[k])}{dx[k]} \sin\left(\frac{2\pi ki}{N} + \theta_i\right); i \in \{1, \dots, N/2 - 1\} \end{aligned}$$

6. The method of claim 1 wherein the gradient vector is computed only as a function of the magnitude of the sample values.

7. The method of claim 1 wherein computing a net penalty function value comprises adding together the individual penalty function values computed for the samples having magnitudes exceeding the predetermined threshold to generate the net penalty function value.

8. In a multicarrier communication system in which a signal to be transmitted comprises data bits to be converted into a symbol modulated by each subcarrier of the signal prior to transmission on a channel, for a signal having a single peak in a frame, a method for minimizing a peak to average power ratio while minimizing introduction of errors into the signal to be transmitted:

sampling the symbols to be transmitted of the frame;

compare magnitudes of the samples of the frame to a predetermined threshold to

determine whether sample magnitudes in the frame violate the

9 predetermined threshold, the predetermined threshold being selectable to
10 control the number of samples violating the threshold;
11 determining an upper limit correction value for each symbol, the upper limit
12 correction value being selectable to control an amount of signal to noise
13 ratio deterioration;
14 computing a peak reduction kernel responsive to the upper limit correction values;
15 responsive to determining a sample magnitude does violate the predetermined
16 threshold, applying the peak reduction kernel to the sample to reduce the
17 peak of the frame; and
18 transmitting the modified symbol.

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5 determining whether the peak of the kernel has a sign equal to a sign of the peak
6 of the frame;
7 responsive to the signs of the peaks of the kernel and the frame being equal,
8 multiplying the peak of the frame by minus one; and
9 adding the peak of the kernel to the samples to reduce the peak of the frame.

1 11. The method of claim 7 in a system in which more than one peak may be present per
2 frame, comprising the steps of:

3 responsive to determining that a sample magnitude exceeds the predetermined
4 threshold, applying the peak kernel to the sample, wherein the peak kernel
5 applied for each sample has a magnitude scaled relative to an extent the
6 sample magnitude exceeds the predetermined threshold.

7 12. The method of claim 11 wherein the scaling factors are chosen to ensure a sum of the
8 magnitudes of the kernels applied is equal to one.